

Exercise 1

$$\det \begin{pmatrix} 7 & 11 \\ -8 & 4 \end{pmatrix} = \begin{vmatrix} 7 & 11 \\ -8 & 4 \end{vmatrix} = 7 \times 4 - 11 \times (-8) = 116$$

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$$\det \begin{pmatrix} 7 & -8 \\ 11 & 4 \end{pmatrix} = \begin{vmatrix} 7 & -8 \\ 11 & 4 \end{vmatrix} = 7 \times 4 - (-8) \times 11 = 116$$

$$\begin{vmatrix} +1 & -0 & +6 \\ -3 & +4 & -15 \\ +5 & -6 & 21 \end{vmatrix} = +1 \begin{vmatrix} 4 & 15 \\ 6 & 21 \end{vmatrix} - 0 \begin{vmatrix} 3 & 15 \\ 5 & 21 \end{vmatrix} + 6 \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix}$$
$$= 1 \times (4 \times 21 - 15 \times 6) - 0 + 6(3 \times 6 - 4 \times 5)$$
$$= -6 - 12 = -18.$$

$$\begin{vmatrix} +1 & -0 & +6 \\ -3 & +4 & -15 \\ +5 & -6 & 21 \end{vmatrix} = -0 \begin{vmatrix} 3 & 15 \\ 5 & 21 \end{vmatrix} + 4 \begin{vmatrix} 1 & 6 \\ 5 & 21 \end{vmatrix} - 6 \begin{vmatrix} 1 & 6 \\ 3 & 15 \end{vmatrix}$$
$$= 4 \times (-9) - 6 \times (-3) = -18$$

$$\begin{vmatrix} 1 & 0 & 6 \\ 3 & 4 & 15 \\ 5 & 6 & 21 \end{vmatrix} = \det(A, B, C) = \det(A, B, C - 6A)$$

$$V(a, b, c) = V(a, b + 5a, c) \quad \text{or}$$

$$V(a, b + 5a, c) = V(a, b, c) + \underbrace{V(a, 5a, c)}_{5V(a, a, c)}_0$$

$$\begin{vmatrix} 1 & 0 & 6 \\ 3 & 4 & 15 \\ 5 & 6 & 21 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 6 - 6 \times 1 \\ 3 & 4 & 15 - 6 \times 3 \\ 5 & 6 & 21 - 6 \times 5 \end{vmatrix} = \begin{vmatrix} 1^+ & 0^- & 0^+ \\ 3^- & 4^+ & -3^- \\ 5^+ & 6^- & -9^+ \end{vmatrix}$$

$$= +1 \times \begin{vmatrix} 4 & -3 \\ 6 & -9 \end{vmatrix} = -36 + 18 = -18$$

$$\begin{aligned} \bullet \begin{vmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{vmatrix} &= \det(A, B, \frac{1}{3}C) = \frac{1}{3} \det(A, B, C) \\ &= \frac{1}{3} \times (-18) = -6 \end{aligned}$$

$$\bullet \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix} = \det(A, B, C, D) = \det(A+B+C+D, B, C, D)$$

$$\begin{vmatrix} 6 & 1 & 2 & 3 \\ 6 & 2 & 3 & 0 \\ 6 & 3 & 0 & 1 \\ 6 & 0 & 1 & 2 \end{vmatrix} = 6 \begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 1 & 3 & 0 & 1 \\ 1 & 0 & 1 & 2 \end{vmatrix} \begin{matrix} L_1 - L_4 \\ L_2 - L_4 \\ L_3 - L_4 \\ L_4 \end{matrix} = 6 \begin{vmatrix} 0^+ & 1 & 1 & 1 \\ 0^- & 2 & 2 & -2 \\ 0^+ & 3 & -1 & -1 \\ 1^- & 0 & 1 & 2 \end{vmatrix}$$

$$= 6 \times -1 \times \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & -1 \\ 3 & -1 & -1 \end{vmatrix} \begin{matrix} C_1 - C_3 \\ C_2 - C_3 \\ C_3 \end{matrix} = -6 \begin{vmatrix} 0^+ & 0^- & 1^+ \\ 3 & 3 & -1 \\ 4 & 0 & -1 \end{vmatrix} = -6 \times +1 \times \begin{vmatrix} 3 & 3 \\ 4 & 0 \end{vmatrix} \\ = -6 \times (-12) = 72$$