Structures fondamentales . TD3 - groupe 1

Equation cartésienne : $\boxed{ax+by+cz = d \quad (1)}$

Soit $M_0=(x_0,y_0,z_0)\in(P)$ alors $\longrightarrow$ $\boxed{ax_0+by_0+cz_0 = d}$ (2)

Donc $a\underbrace{(x-x_0)}_{u}+b\underbrace{(y-y_0)}_{v}+c\underbrace{(z-z_0)}_{w}=0$ $\big((1)-(2)\big)$

$$\boxed{a\,u + b\,v + c\,w = 0}$$

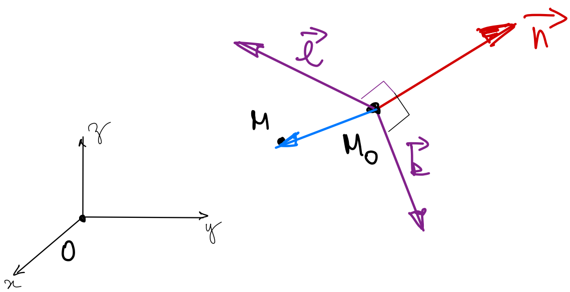
$\iff (a,b,c)\cdot(u,v,w)=0 \iff (u,v,w)\perp \overbrace{(a,b,c)}^{\vec{n}}$

$$\begin{cases} u = x-x_0 \\ v = y-y_0 \\ w = z-z_0 \end{cases} \iff \begin{cases} x = x_0+u \\ y = y_0+v \\ z = z_0+w \end{cases}$$

$M=(x,y,z)$

$M_0=(x_0,y_0,z_0)$

$\overrightarrow{M_0M}=(u,v,w)$



$\boxed{\overrightarrow{M_0M}\in \text{vect}(\vec{k},\vec{\ell})}$ $\longrightarrow$ équation paramétrique

il existe $s,t\in\mathbb{R}$ tels que $\overrightarrow{M_0M}=s\vec{k}+t\vec{\ell}$

$\vec{k} = (x_k, y_k, z_k)$ $\quad \vec{\ell} = (x_\ell, y_\ell, z_\ell)$

$$\boxed{\begin{cases} x - x_0 = s x_k + t x_\ell \\ y - y_0 = s y_k + t y_\ell \\ z - z_0 = s z_k + t z_\ell \end{cases}, \quad s,t \in \mathbb{R}.}$$

Exercice 4 Equation du plan (P) contenant les points

$A = (1,1,1)$ $\quad B = (2,0,1)$, $\quad C = (-1,2,4)$

Qui est $\vec{n}$?

Si on connait $\vec{k}$ et $\vec{\ell}$ alors $\vec{n} = \vec{k} \wedge \vec{\ell}$ fait l'affaire

c'à d. $\vec{n} \perp (\vec{k}, \vec{\ell})$

On prend $\vec{k} = \vec{AB}$, $\quad \vec{\ell} = \vec{AC}$

$\vec{AB} = (x_B - x_A, y_B - y_A, z_B - z_A) = (1,-1,0) = \vec{k}$

$\vec{AC} = (-2,1,3) = \vec{\ell}$

Equation paramétrique. $(x_0, y_0, z_0) = (x_A, y_A, z_A) = (1,1,1)$

$(x_k, y_k, z_k) = (1,-1,0)$ $\qquad (x_\ell, y_\ell, z_\ell) = (-2,1,3)$

$$\boxed{\begin{cases} x - 1 = s - 2t \\ y - 1 = -s + t \\ z - 1 = 3t \end{cases}, \quad s,t \in \mathbb{R}}$$

Equation cartésienne $ax + by + cz = d$

$$\vec{k} \wedge \vec{l} = (-3, -3, -1) \longrightarrow (3, 3, 1)$$

$$3x + 3y + z = d$$

$$\boxed{3x + 3y + z = 7}$$

$$d = 3x_A + 3y_A + z_A = 7$$
$$= 3x_B + 3y_B + z_B$$

Exercice 5

$$(1) \begin{cases} x = 2 + s + 2t \\ y = 2 + 2s + t \\ z = 1 - s - t \end{cases} \qquad \begin{cases} x = 1 + 3s' - t' \\ y = 3 + 3s' + t' \\ z = 1 - 2s' \end{cases} (2)$$

$$\vec{k} = (1, 2, -1) \qquad \vec{k}' = (3, 3, -2)$$
$$\vec{l} = (2, 1, -1) \qquad \vec{l}' = (-1, 1, 0)$$

$$\vec{k} \wedge \vec{l} \qquad \vec{k}' \wedge \vec{l}'$$

$$\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ -1 & -1 \end{vmatrix} \longrightarrow \begin{pmatrix} -1 \\ -1 \\ -3 \end{pmatrix} \qquad \begin{vmatrix} 3 & -1 \\ 3 & 1 \\ -2 & 0 \end{vmatrix} \longrightarrow \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix}$$

$$\times(-2)$$

$$(2, 2, 1) \in (1)$$

On injecte $(x,y,z)=(2,2,1)$ dans ②.

$$\begin{cases} 2 = 1+3s'-t' \\ 2 = 3+3s'+t' \\ 1 = 1-2s' \end{cases} \Longleftrightarrow \begin{cases} 3s'-t' = 1 \\ 3s'+t' = -1 \\ s' = 0 \end{cases}$$

$$\Longleftrightarrow \begin{cases} s'=0 \\ t'=-1 \end{cases}$$