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P. CATTIAUX
C. LÉONARD

Erratum Correction to: Minimization of the Kullback information of diffusion processes

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ERRATUM

Correction to:

Minimization of the Kullback information of diffusion processes (*)

by

P. CATTIAUX and C. LÉONARD

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The supermartingale Z defined by (1.11) does not vanish for $t > T_\infty$, if ω belongs to the set $A = \bigcup_{m=1}^{\infty} \{T_m = T_\infty < T\}$, contrary to what is written at the bottom of page 90. Of course the associated Föllmer-measure Q and the initial P are equivalent in restriction to A, so that if $Q(T_\infty < T) = 0$, then $P(A) = 0$. This holds if we know that Q is a probability measure on Ω such that $Q(T_\infty < T) = 0$.

As a consequence, we cannot conclude in Proposition (2.3) that $T_\infty = T, Q$-a.s. $\left(\text{since on } A, \int_0^{T_\infty} \beta_s \cdot a(X_s, s)\beta_s \, ds < +\infty\right)$, the rest of the proposition being true. So the trick (2.6) is incomplete, and to both the energy condition and the domination condition, one has to add the third condition: $Q(T_\infty < T) = 0$.

This additional condition is checked in the proof of Theorem (3.1), thanks to (3.47), so that the proof of the theorem is complete.

(*) Vol. 30, n° 1, 1994, pp. 83-132.

In order to complete the proofs in sections 4 and 5, we have to check that this condition is satisfied. We shall present this below, in the context of Theorem (4.18), but the proof still works for any other case considered in the paper.

Denote

$$A_m = \{T_m = T_\infty < T\} = \bigcap_{j\geq 1} A_m^j,$$

where $A_m^j = \left\{ \int_{T_m}^{T_m+1/j} (BaB)(X_s, s)\, ds = +\infty \right\}$,

$$A_m^{j,k} = \left\{ \int_{T_m}^{T_m+1/j} [(BaB)(X_s, s) \wedge k]\, ds > j \right\}$$

and

$$A_m^{j,\infty} = \bigcup_{k\geq 1} A_m^{j,k}.$$

One clearly has $A_m^{j,\infty} \supseteq A_m^j \supseteq A_m$ for all j.

Now, denote $P(A_m) = \alpha \geq 0$. We are going to build a subset B_m of A_m, with a positive mass and which is an intersection of $A_m^{j,k}$. To this end, choose k_1 such that $P(A_m \cap A_m^{1,k_1}) \geq \alpha(1 - 1/2)$. This is possible because $A_m^{1,k}$ is an increasing sequence of sets such that $A_m^{1,\infty} \supseteq A_m$. We put $B_m^1 = A_m \cap A_m^{1,k_1} \subset A_m$, and we can choose k_2 such that $P(B_m^1 \cap A_m^{2,k_2}) \geq \alpha(1 - 1/2)(1 - 1/4)$, by the same argument as above. Now, by induction we build a nonincreasing sequence B_m^j of subsets such that

$$B_m^j \subset A_m \cap \left(\bigcap_{i=1}^{j} A_m^{i,k_i} \right) \quad \text{and} \quad P(B_m^j) > \alpha \prod_{i=1}^{j} (1 - 2^{-i}).$$

Finally $B_m = \bigcap_{j\geq 1} B_m^j$ satisfies $B_m \subset A_m \cap \left(\bigcap_{i=1}^{\infty} A_m^{i,k_i} \right)$ and $P(B_m) \geq c\alpha \geq 0$, with $c = \prod_{i=1}^{\infty} (1 - 2^{-i}) > 0$.

But for each $n \geq 1$, P and Q^n are equivalent measures so that

$$P(B_m) = E^{Q^n}\left[\mathbb{1}_{B_m} (Z_T^n)^{-1} \right] = E^{Q^n}\left[\mathbb{1}_{B_m} (Z_{T_m}^n)^{-1} \right]$$

since B_m is $\mathcal{F}_{T_m^+}$-measurable, which yields for all $n \geq 1$

$$(\star) \quad P(B_m) \leq (Q^n(B_m))^{1/2} \left(E^{Q^n}\left[(Z_{T_m}^n)^{-2} \right] \right)^{1/2} \leq (Q^n(B_m))^{1/2} e^{m/2}$$

since $\int_0^{T_m} B^n a B^n(X_s, s)\, ds \leq \int_0^{T_m} BaB(X_s, s)\, ds \leq m$. Now, we can evaluate $Q^n(B_m)$ by means of (4.26). Indeed, for $\varepsilon, k, M > 0$

$$\begin{aligned}
&Q^n\left(\int_{T_m}^{T_m+\varepsilon} BaB \wedge k\, ds > M\right)\\
&\leq Q^n\left(\int_0^T BaB \wedge k\, ds > M\right)\\
&\leq \frac{1}{M} E^{Q^n}\left[\int_0^T BaB \wedge k\, ds\right]\\
&\leq \frac{1}{M}\Big(\|B\|^2_{L^2_\nu} + 2kT\|B - B^n\|_{L^2_\nu}(2 + \|B - B^n\|^2_{L^2_\nu})^{1/2}\Big).
\end{aligned}$$

It follows that for a fixed i,

$$\inf_{n\geq 1} Q^n\left(A_m^{i,k_i}\right) \leq \frac{1}{i}\|B\|^2_{L^2_\nu},$$

hence $\inf_{n\geq 1} Q^n(B_m) = 0$, which is in contradiction with $(\star)$ unless $\alpha = 0$. We thus have proved that $P(A_m) = 0$, hence$P(A) = 0$.

Consequently, all the results of the paper are true, except for Proposition (2.3). All our apologies.